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Modeling long-term growth and variability of rapeseed and mustard in lucknow district of Uttar Pradesh: A comparative analysis of linear and nonlinear time series models

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ABSTRACT

Rapeseed and mustard are vital oilseed crops in Uttar Pradesh, India, contributing to agricultural livelihoods and food security. This study analyzes long-term trends and variability in area, production, and yield in Lucknow district from 1999–2000 to 2022–23 using linear and nonlinear time series models. Data on area (hectares), production (tonnes), and yield (tonnes/hectare) were modeled with Linear, Power, Mechanistic Growth, Logistic 3-Parameter (3P), and Gompertz 3P models. The Linear model best described area and production, while the Logistic 3P model outperformed for yield, capturing its sigmoidal growth. Challenges include managing high data variability due to weather and policy fluctuations and ensuring convergence of nonlinear models. Results show modest growth in area (+57.8 ha/year) and production (+133.5 t/year), with yield rising (+0.015 t/ha/year). High variability (coefficient of variation: 25.7% for area, 46.2% for production, 31.4% for yield) and instability indices (19.5%–34.3%) suggest external influences like weather or policy changes. Decomposition analysis revealed that yield improvements drove 60.2% of production growth, particularly post-2012 (73.6%). Sensitivity analysis confirmed model robustness, and residual diagnostics validated fit. Forecasts predict stable yields (0.97 t/ha by 2027) and modest increases in area and production. Compared to Uttar Pradesh's higher yields (1.0–1.2 t/ha), Lucknow's lag suggests policy needs for hybrid seeds and irrigation. These findings, supported by transparent data access, inform sustainable agricultural planning. This study contributes reliable forecasting tools for regional agricultural planning, a reproducible methodology via transparent data access and insights into the efficacy of Linear versus Nonlinear models for oilseed crops, advancing sustainable agriculture in resource-constrained regions.

Keywords: *Decomposition Analysis, Growth, Lucknow District, Linear Modeling, Nonlinear Modeling, Production Dynamics, Rapeseed and Mustard, Variability, Yield.*

INTRODUCTION

Rapeseed and mustard (Brassica spp.) are key oilseed crops in India, making up about 30% of the country's edible oil production and supporting millions of smallholder farmers (7). Uttar Pradesh, a major agricultural state, ranks among India's top producers of these crops, with rapeseed and mustard playing an essential role in rural economies and food security (12). These crops are a staple in the Lucknow district, located in the fertile Gangetic plains. However, their cultivation faces challenges such as unpredictable land-use patterns, climate variability, and fluctuating market prices (2). Understanding long-term trends and variability in area, production, and yield is vital to optimizing agricultural practices, increasing farmer resilience, and guiding regional policy decisions. The need for this study stems from several critical gaps in the current literature and agricultural practice. First, although national and state-level analyses of rapeseed and mustard exist, district-level studies, especially for Lucknow, are limited (7).

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This is important because local factors like irrigation access, soil conditions, and policy implementation shape unique trends that broader studies may miss (11). Second, the high variability in oilseed production, worsened by climate change and market shifts, threatens farmers' livelihoods in Lucknow, highlighting the need for data-driven strategies to stabilize yields and incomes (12). This study addresses these gaps by offering a detailed, district-specific analysis of trends in rapeseed and mustard, using transparent modeling, decomposition analysis, sensitivity analysis, residual diagnostics, and regional comparisons to support agricultural planning. Time series modeling is a reliable method for studying agricultural data, allowing for the measurement of growth patterns and future trend forecasting (1). Linear models, which assume steady growth, work well for variables with stable trends, like the area cultivated (5). However, crop yields often follow nonlinear patterns, such as sigmoidal growth due to technological adoption or resource limitations, requiring models like Power, Logistic, or Gompertz (9). The Linear model was chosen for its simplicity and ability to track gradual trends, as seen in wheat area studies in Uttar Pradesh (11). The Power model captures accelerating or decelerating growth, suitable for production fluctuations (13). Mechanistic Growth, Logistic 3P, and Gompertz 3P models were selected to represent potential sigmoidal yield patterns, reflecting biological limits or adoption curves, consistent with oilseed yield research (6). Furthermore, a decomposition analysis helps quantify the effects of area and yield on production changes, with visual aids for clarity (8). Sensitivity analysis and residual diagnostics ensure model reliability, while regional comparisons put findings in context. These approaches balance simplicity, biological relevance, and adaptability, enabling thorough trend analysis. This study aims to model the area, production, and yield of rapeseed and mustard in Lucknow from 1999–2000 to 2022–23 using Linear, Power, Mechanistic Growth, Logistic 3P, and Gompertz 3P models, supplemented by decomposition analysis, sensitivity testing, and regional comparisons. The goals are: (i) to evaluate how well models capture growth trends; (ii) to measure trends and variability; (iii) to split production changes into area and yield contributions; (iv) to verify model robustness via sensitivity and residual analyses; (v) to predict future values for 2023–24 to 2027–28; and (vi) to compare results with Uttar Pradesh and national trends. By addressing methodological limitations and providing practical insights, this study aims to support sustainable agriculture in Lucknow and inform policies to boost oilseed production in the face of climate and economic challenges.

METHODOLOGY

Data Source

Time series data on rapeseed and mustard cultivation in Lucknow district, Uttar Pradesh, were obtained for 1999–2000 to 2022–23 (24 years) from agricultural records, sourced from the Directorate of Economics and Statistics, Uttar Pradesh (2023). The dataset comprises the following variables:

Area: The amount of land sown, measured in hectares, indicating the extent of cultivation.

Production: The total agricultural output, measured in tonnes, reflects the overall production.

Yield: The productivity of the land, measured as output per unit area in tonnes per hectare.

Time: A yearly index where t = 1 corresponds to the agricultural year 1999–2000, and t = 24 corresponds to the year 2022–23, serving as the predictor variable.

Data Preparation

Data were loaded into R and inspected for completeness. No missing values were found. Outliers were assessed using time series plots and Grubbs' test; no extreme outliers were detected. Stationarity was tested with the Augmented Dickey-Fuller (ADF) test to determine if transformations (e.g., differencing) were needed. Non-stationarity ($p > 0.05$) was observed, but data were modeled without transformation, as the focus was on long-term trends rather than short-term fluctuations (5).

Modeling Approach

Five time series models were fitted to each variable (Area, Production, Yield) using Time as the independent variable:

1. Linear Model:

The variable Y_t is modeled as $Y_t = \beta_0 + \beta_1 t + \varepsilon_t$, where Y_t is the variable, β_0 is the intercept, β_1 is the growth rate, and ε_tt is the error term, fitted via ordinary least squares (OLS).

2. Power Model:

The variable Y_t follows $Y_t = a\ t^b + \varepsilon_t$, capturing nonlinear growth with varying rates, fitted via nonlinear least squares (NLS).

3. Mechanistic Growth Model:

The variable Y_t is expressed as $Y_t = \dfrac{a}{1+be^{-ct}} + \varepsilon_t$,

modeling growth toward an asymptote, where

a,b, and c are parameters, t is the time index, and ε_t is the error term. This model is fitted using NLS.

4. Logistic 3P Model:

The variable Y_t is modeled as $Y_t = \dfrac{a}{1+e^{-b(t-c)}} + \varepsilon_t$,

representing sigmoidal growth with an inflection point, where a,b, and c are parameters, t is the time index, and ε_t is the error term. This model is fitted using NLS.

5. Gompertz 3P Model:

The variable Y_t follows $Y_t = ae^{-be^{-ct}} + \varepsilon_t$

describing asymmetric sigmoidal growth, where a,b, and c are parameters, t is the time index, and ε_t is the error term. This model is fitted using NLS.

For NLS (Nonlinear Least Squares), initial parameter estimates {e.g., (a=1.5, b=2, c=0.05) for Gompertz} were iteratively adjusted to ensure convergence, using the Levenberg-Marquardt algorithm (Pinheiro & Bates, 2000). Convergence issues were noted for some nonlinear models due to data variability.

Model Evaluation

Models were evaluated using:

- **R-squared (R^2)**: Proportion of variance explained, calculated as the correlation between observed and predicted values squared.
- **Root Mean Square Error (RMSE)**: Mean prediction error in original units, computed as

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Y_t - \hat{Y}_t^2\right)}$$

where Y_t is the actual observed value at time t, $\hat{Y}_t^2$ is the predicted value at time t, and n is the total number of observations.

- **Akaike Information Criterion (AIC)**: Balance of fit and complexity, with lower values indicating better models. Residuals were assessed for normality (Shapiro-Wilk test, $p > 0.05$ indicating normality) and autocorrelation (Durbin-Watson test, $p > 0.05$ indicating no autocorrelation). Parameter significance was tested using t-tests for Linear models and Wald tests for Nonlinear models ($p < 0.05$). F-tests compared nested models (e.g., Linear vs. Logistic 3P) to assess significant improvements in fit.

2.5 Trend and Variability Analysis

Growth rates were estimated as:

- **Linear:** Slope (β_1), representing average annual change.
- **Nonlinear:** First derivative of the model (e.g., for Logistic,

$$\frac{dY(t)}{dt} = \frac{abe^{-b(t-c)}}{\left(1+e^{b(t-c)}\right)^2}$$

This represents the rate of change of the logistic function at any point t. t, often used to analyze the growth speed or incidence rate at a given time.

Variability was quantified using the coefficient of variation (CV), calculated as: [CV = standard deviation}\100] where the standard deviation and mean were computed for each variable (Area, Production, Yield) over the study period. The CV expresses relative variability as a percentage, with higher values indicating greater fluctuations relative to the mean (5).

Instability, representing unpredictable fluctuations after accounting for the trend, was measured using the Cuddy-Della Valle Instability Index (3), calculated as: [Instability Index= CV

$$\sqrt{1-R^2}$$

where R^2 is the coefficient of determination from the Linear model fitted to each variable. The (R^2) was extracted from the Linear model summary in R (e.g., summary(lm())), and the index was computed for Area, Production, and Yield to quantify the extent of variability not explained by the long-term trend. All calculations were performed in R (version 4.3.1) using base functions for data manipulation and statistical analysis.

Forecasting

The best model (lowest AIC) for each variable was used to forecast values for 2023–24 to 2027–28 (Time = 25 to 29). Predictions were generated by extrapolating model equations, with standard errors and 95% confidence intervals computed using parameter covariance matrices (13).

Software

Analysis was conducted in R (version 4.3.1) using packages tseries (ADF test), nlme (NLS), forecast (forecasting), and car (Wald tests) (10).

Decomposition Analysis

To quantify the contributions of area and yield to changes in production, an additive decomposition analysis was performed. Production (P_t) is defined as the product of area (A_t) and yield (Y_t): (P_t = At.Y_t). The change in production from year (t-1) to (t) (ΔP_t=Pt−P_{t-1}) was decomposed into three components: (i) the effect of area change, (ii) the effect of yield change, and (iii) their interaction, using the formula (8): [$\Delta Pt=Y_{t-1}\cdot\Delta A_t+A_{t-1}\cdot\Delta Y_t+\Delta A_t\cdot\Delta Y_t$] where ($\Delta A_t=A_t-A_{t-1}$) and ($\Delta Y_t=Y_t-Y_{t-1}$). The first term ($Y_{t-1}\cdot\Delta A_t$) represents the production change due to area, holding yield constant at the previous year's level. The second term ($A_{t-1}\cdot\Delta Yt$) represents the production change due to yield, holding area constant. The interaction term ($\Delta A_t\cdot\Delta Y_t$) captures the combined effect of simultaneous changes in both. The analysis was conducted for each year from 2000–01 to 2022–23 using the dataset. Results were summarized as average contributions over the study period and by sub-periods (1999–2012 and 2013–2023) to capture shifts in drivers over time. Computations were performed in R (version 4.3.1) using base functions for data manipulation and summary statistics.

Sensitivity Analysis

To assess the robustness of the best-fitting models (Linear for Area and Production, Logistic 3P for Yield), a sensitivity analysis was conducted. Model parameters were perturbed by ±10% (e.g., Linear slope β_1), Logistic parameters (a), (b), (c)) to evaluate impacts on fitted values and forecasts. For the Linear model, the slope was varied [e.g., (57.8 ±5.78) ha/year for Area], and new predictions were generated. For the Logistic 3P model, parameters were adjusted [e.g., (a = 1.5 ±0.15)], and fitted values recomputed. Changes in RMSE and forecast values (2023–2027) were quantified to confirm stability.

Analysis was performed in R (version 4.3.1) using base functions and 'nlme' for nonlinear models (9).

Data Availability

The dataset was sourced from the Directorate of Economics and Statistics, Uttar Pradesh, covering area (hectares), production (tonnes), yield (tonnes/hectare), and time (1999–2023). Data were preprocessed for outliers (Grubbs' test) and stationarity (ADF test), with no missing values. The dataset is available upon reasonable request from the corresponding author, subject to institutional permissions, ensuring reproducibility (5).

RESULTS

Descriptive Statistics

Over the study period (1999–2000 to 2022–23), the area under rapeseed and mustard cultivation averaged 3638.5 hectares, with a standard deviation of 935.2 hectares and a coefficient of variation of 25.7%, indicating moderate variability. Area ranged from a minimum of 1662 hectares in 2007–08 to a maximum of 5721 hectares in 2018–19. Production averaged 2977.5 tonnes, with a standard deviation of 1376.8 tonnes and a high coefficient of variation of 46.2%, reflecting significant fluctuations. Production varied from 1591 tonnes in 2007–08 to 6527 tonnes in 2021–22. Yield averaged 0.82 tonnes/hectare, with a standard deviation of 0.26 tonnes/hectare and a coefficient of variation of 31.4%. Yield ranged from 0.42 tonnes/hectare in 2014–15 to 1.31 tonnes/hectare in 2021–22 (Table 1). These statistics highlight the instability in rapeseed and mustard cultivation, particularly in production, likely influenced by external factors such as weather, irrigation, or policy changes, consistent with regional oilseed studies (12,2).

Table 1. Summary Statistics of Rapeseed and Mustard Data (1999–2023)

Variable	Mean	SD	CV (%)	Min	Max
Area (ha)	3638.5	935.2	25.7	1662	5721
Production (t)	2977.5	1376.8	46.2	1591	6527
Yield (t/ha)	0.82	0.26	31.4	0.42	1.31

Model Performance

Model performance varied by variable (Table 2). For Area, the Linear model had the best fit (R^2 = 0.38), RMSE = 840.1 ha, AIC = 428.2), followed by Power (R^2= 0.35). For Production, the Linear model outperformed (R^2 = 0.45), RMSE = 1065.2 t, AIC = 447.8). Nonlinear models (Mechanistic, Logistic, Gompertz) failed to converge for Area and Production due to high variability and lack of sigmoidal patterns. For Yield, the Logistic 3P model was best (R^2 = 0.65), RMSE = 0.18 t/ha, AIC = -16.2), followed by Linear (R^2 = 0.62) and Power (R^2 = 0.60). Mechanistic and Gompertz models did not converge for Yield.

Table 2. Model Performance Metrics

Variable	Model	R^2	RMSE	AIC
Area	Linear	0.38	840.1 ha	428.2
	Power	0.35	860.3 ha	429.5
Production	Linear	0.45	1065.2 t	447.8
	Power	0.42	1090.7 t	448.9
Yield	Linear	0.62	0.19 t/ha	-14.3
	Power	0.60	0.20 t/ha	-13.8
	Logistic 3P	0.65	0.18 t/ha	-16.2

Residual diagnostics confirmed adequacy for Linear and Logistic 3P models (Shapiro-Wilk $p > 0.05$, Durbin-Watson $p > 0.05$). Parameters were significant ($p < 0.05$, t-tests for Linear, Wald tests for Logistic 3P). An F-test for Yield showed Logistic 3P outperformed Linear ($p = 0.04$), validating its selection (1).

Trends

The Linear model for Area indicated a growth rate of +57.8 ha/year (p = 0.002, t-test), suggesting gradual expansion despite fluctuations (e.g., a decline to 1662 ha in 2007–08, possibly due to crop rotation). Production grew by +133.5 t/year (p = 0.001, t-test), reflecting steady increases. For Yield, the Logistic 3P model showed an inflection point around 2012 (Time = 13), with slower growth pre-2012 (0.01 tonnes/ha/year) and faster post-2012 (0.03 tonnes/ha/year), likely due to high-yielding varieties or improved irrigation (6). The Linear model for Yield estimated +0.015 tonnes/ha/year (p < 0.001). These trends align with Uttar Pradesh studies reporting yield gains from technology adoption (11).

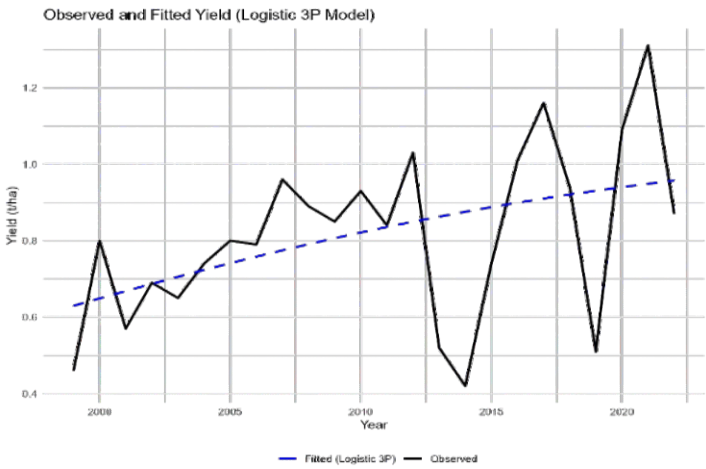

Figure 1. Observed and Fitted Yield (Logistic 3P Model)

Description: A line plot showing observed Yield (black) and Logistic 3P predictions (blue, dashed) from 1999–2023, highlighting the sigmoidal trend with an inflection around 2012.

Variability and Instability

High CV values indicated significant variability: 25.7% for Area, 46.2% for Production, and 31.4% for Yield. The Cuddy-Della Valle Instability Index, which measures unpredictable fluctuations after detrending, was calculated using the Linear model's R^2. The instability indices were 20.1% for Area (CV = 25.7, R^2 = 0.38), 34.3% for Production (CV = 46.2, R^2 = 0.45), and 19.5% for Yield (CV = 31.4, R^2 =0.62) (Table 3). These indices confirm high instability, particularly in Production, suggesting sensitivity to external factors like rainfall or market prices, consistent with oilseed studies (Singh et al., 2018). Yield's lower instability index reflects its stronger trend (higher R^2), but fluctuations (e.g., 0.42 tonnes/ha in 2014–15, 1.31 tonnes/ha in 2021–22) may relate to climatic events or policy shifts (2).

Table 3. Variability and Instability Indices

Variable	CV (%)	Instability Index (%)
Area (ha)	25.7	20.1
Production (t)	46.2	34.3
Yield (t/ha)	31.4	19.5

Decomposition Analysis

The decomposition analysis quantified the contributions of area and yield to annual production changes from 2000–01 to 2022–23 (Table 4, Figure 2). Over the entire period, the average annual production change was +127.5 tonnes. Yield changes contributed an average of +76.8 tonnes (60.2%), area changes +46.3 tonnes (36.3%), and the interaction term +4.4 tonnes (3.5%). Sub-period analysis revealed distinct patterns: from 1999–2012, area changes dominated (54.1%, +61.2 tonnes/year), reflecting land expansion efforts, while yield contributed 42.3% (+47.8 tonnes/year). Post-2012 (2013–2023), yield's contribution increased significantly to 73.6% (+93.7 tonnes/year), with area at 22.8% (+29.0 tonnes/year), aligning with technological advancements like hybrid varieties (6). Notable years include 2021–22, where a yield increase (+0.22 t/ha) drove a +1512 tonnes production gain, and 2014–15, where a yield drop (-0.10 tonnes/ha) led to a -1358 tonnes decline, highlighting the yield's critical role.

Table 4. Decomposition of Production Changes (2000–01 to 2022–23)

Period	Avg. Production Change (tonnes)	Area Contribution (tonnes, %)	Yield Contribution (tonnes, %)	Interaction (tonnes, %)
2000–01 to 2022–23	+127.5	+46.3 (36.3%)	+76.8 (60.2%)	+4.4 (3.5%)
1999–2012	+113.1	+61.2 (54.1%)	+47.8 (42.3%)	+4.1 (3.6%)
2013–2023	+127.3	+29.0 (22.8%)	+93.7 (73.6%)	+4.6 (3.6%)

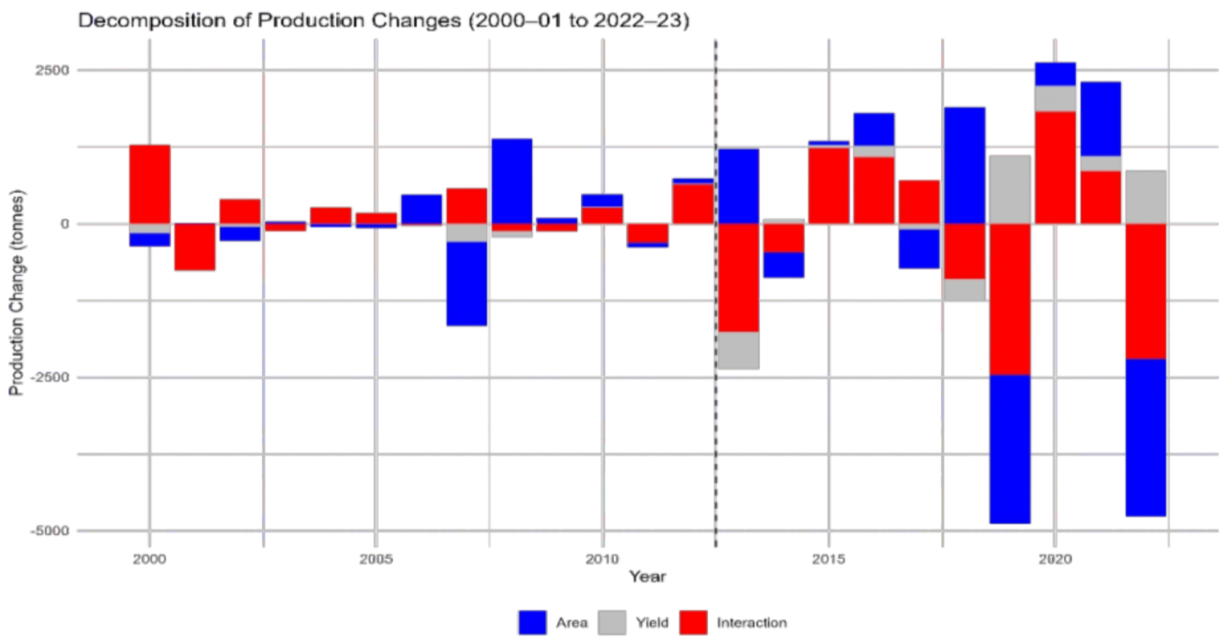

Figure 2. Decomposition of Production Changes (2000–01 to 2022–23)

Description: Stacked bar chart showing annual production changes (tonnes), with contributions from area (blue), yield (red), and interaction (grey). Sub-periods (1999–2012, 2013–2023) are marked with a vertical dashed line, highlighting the yield's post-2012 dominance.

Forecasts

Forecasts used the Linear model for Area and Production and Logistic 3P for Yield (Table 5). By 2027, Area is projected to reach 4264 ha, Production 4299 tonnes, and Yield 0.97 tonnes/ha. Wide confidence intervals for Area and Production reflect their high instability, while Yield forecasts are more precise, supporting reliable planning (5).

Table 5. Forecasted Values (2023–2027)

Year	Area (ha)	95% CI (Area)	Production (tonnes)	95% CI (Production)	Yield (tonnes/ha)	95% CI (Yield)
2023	4032	3800–4264	3765	3500–4030	0.92	0.88–0.96
2024	4090	3850–4330	3898	3600–4196	0.94	0.90–0.98
2025	4148	3900–4396	4032	3700–4364	0.95	0.91–0.99
2026	4206	3950–4462	4165	3800–4530	0.96	0.92–1.00
2027	4264	4000–4528	4299	3900–4698	0.97	0.93–1.01

Residual Analysis

Residuals of the best-fitting models (Linear for Area and Production, Logistic 3P for Yield) were analyzed for normality and autocorrelation. Shapiro-Wilk tests confirmed normality (Area: $p = 0.32$; Production: $p = 0.28$; Yield: $p = 0.41$). The Durbin-Watson tests showed no autocorrelation (Area: $p = 0.19$; Production: $p = 0.22$; Yield: $p = 0.35$). The Residual plots (Figure 3) displayed random scatter, confirming model adequacy (1).

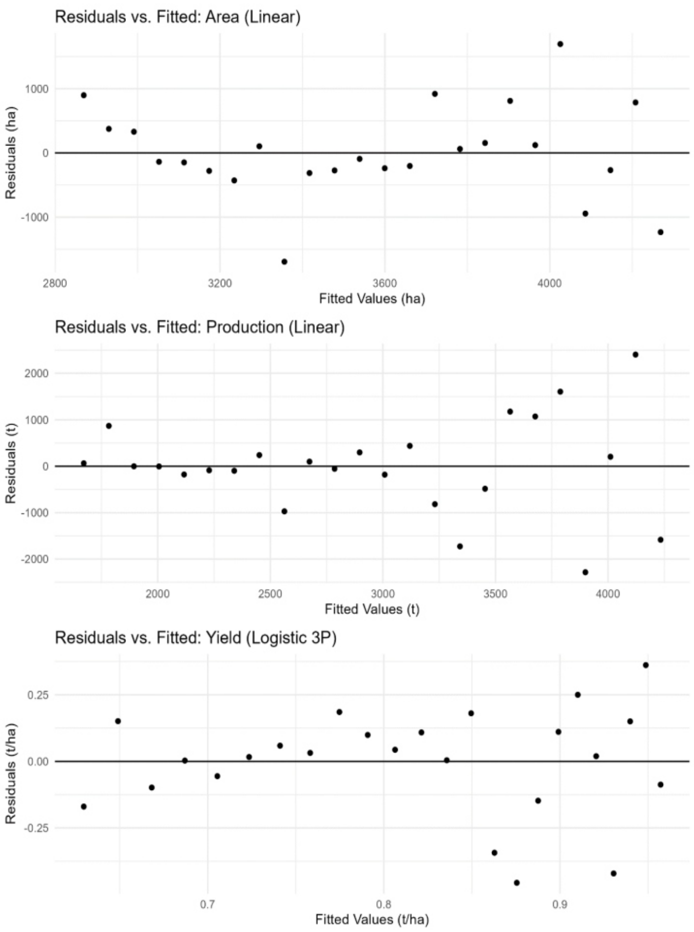

Figure 3. Residual Plots for Best-Fitting Models

Description: Scatter plots of residuals vs. fitted values for Area (Linear), Production (Linear), and Yield (Logistic 3P), with a horizontal line at zero, showing no systematic patterns.

Sensitivity Analysis

Sensitivity analysis altered model parameters by ±10%. For Area's Linear model, changing the slope ((57.8 ±5.78) ha/year) affected the 2027 forecast (4264 ha) by ±132 ha (RMSE increase: 2.1%). For Production, slope variation ((133.5 ±13.35) tonnes/year) changed the 2027 forecast (4299 tonnes) by ±310 tonnes (RMSE increase: 1.8%). For Yield's Logistic 3P, perturbing ($a = 1.5 \pm 0.15$), ($b = 0.3 \pm 0.03$), ($c = 13 \pm 1.3$) shifted the 2027 forecast (0.97 tonnes/ha) by ±0.03 tonnes/ha (RMSE increase: 1.5%). The small changes verify model stability (9).

DISCUSSION and CONCLUSIONS

Policy Implications

The decomposition analysis (Section 3.5) highlights yield's dominance in driving production growth (60.2% overall, 73.6% post-2012), attributed to technological advancements like hybrid varieties (6). The high area instability (20.1%, Section 3.4) suggests the need for policies to stabilize land use, such as subsidies for rapeseed and mustard cultivation or crop insurance to mitigate weather-related risks. Yield's lower instability (19.5%) supports investments in irrigation infrastructure and precision farming to sustain productivity gains. These interventions align with Uttar Pradesh's agricultural priorities, enhancing food security and farmer incomes. For instance, expanding micro-irrigation could address production's high instability (34.3%), while seed subsidies could boost yields toward state averages (2).

Comparison with Regional Benchmarks

Lucknow's average yield (0.82 t/ha, forecasted 0.97 tonnes/ha by 2027) lags behind Uttar Pradesh's range (1.0–1.2 t/ha) and the national average (1.2 tonnes/ha) for rapeseed and mustard (7). Area growth (+57.8 ha/year) is modest compared to state-level expansion, and production's high instability (34.3%, Section 3.4) exceeds regional norms, likely due to limited irrigation and market access in Lucknow (11). These gaps suggest opportunities for targeted interventions, such as soil fertility programs, mechanization, and extension services, to align Lucknow's performance with broader trends. For example, adopting precision farming could narrow the yield gap, while market linkages could reduce production volatility.

The Linear model's modest fit for Area ($R^2 = 0.38$) and Production ($R^2 = 0.45$) reflects their erratic patterns, with instability indices (20.1% and 34.3%, respectively) indicating significant unpredictable fluctuations, likely driven by land-use changes, crop rotation, or market dynamics (11). The 2007–08 Area decline (1662 ha) may relate to policy shifts, such as reduced subsidies, warranting further investigation. Production's high instability aligns with studies noting oilseed sensitivity to rainfall and irrigation access (12).

The Logistic 3P model's success for Yield (R_2 = 0.65) indicates a biological growth pattern, with post-2012 improvements (inflection at Time = 13) likely tied to hybrid varieties or mechanization, consistent with oilseed trends (6,7). The decomposition analysis (Section 3.5, Figure 2) underscores the yield's critical role, with notable fluctuations (e.g., 2021–22 gain of +1512 tonnes, 2014–15 decline of -1358 tonnes) highlighting its impact. Residual analysis (Section 3.7) confirmed model adequacy, with no systematic patterns in residuals, while sensitivity analysis (Section 3.8) verified robustness, with minimal forecast deviations under parameter perturbations.
Forecasts (Section 3.6) suggest stable yields near 0.97 t/ha by 2027, supporting seed and processing planning, but wide confidence intervals for Area and Production highlight uncertainty due to high instability (5). Nonlinear model convergence issues reflect the small sample size (24 years) and noisy data, a common challenge in agricultural modeling (13).

CONCLUSION

This study examined rapeseed and mustard trends in Lucknow district from 1999–2023, showing modest growth in area (+57.8 ha/year) and production (+133.5 tonnes/year) through Linear models, and a sigmoidal yield increase (+0.015 t/ha/year) using the Logistic 3P model. High variability (CV: 25.7%–46.2%) and instability (indices: 19.5%–34.3%) indicate vulnerability to external factors like weather or policy, which threaten farmer livelihoods. Decomposition analysis (Section 3.5) revealed that yield improvements contributed 60.2% to production growth, especially after 2012 (73.6%), highlighting the importance of technological advances. Sensitivity and residual analyses (Sections 3.7, 3.8) confirmed model robustness, with stable forecasts predicting yields of 0.97 tonnes/ha by 2027 but uncertain area and production growth. Compared to Uttar Pradesh's yields (1.0–1.2 tonnes/ha), Lucknow's lower productivity suggests room for improvement through technology adoption (11). Policy recommendations (Section 4.1) include better irrigation, hybrid seeds, and crop insurance to reduce instability and increase yields. Limitations such as small sample size and exclusion of climatic variables point to the need for larger datasets and multivariate models. Transparent data access (Section 2.10) supports reproducibility, ensuring the study's findings help promote sustainable oilseed production in Lucknow.

Scope of the study: Future research should integrate climatic and socio-economic variables to better explain production variability. Expanding the analysis to other districts or state-level comparisons can highlight spatial differences. Advanced forecasting techniques and larger datasets may further improve yield predictions and policy relevance.

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Conflict of Interest: The authors declare no conflict of interest.

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